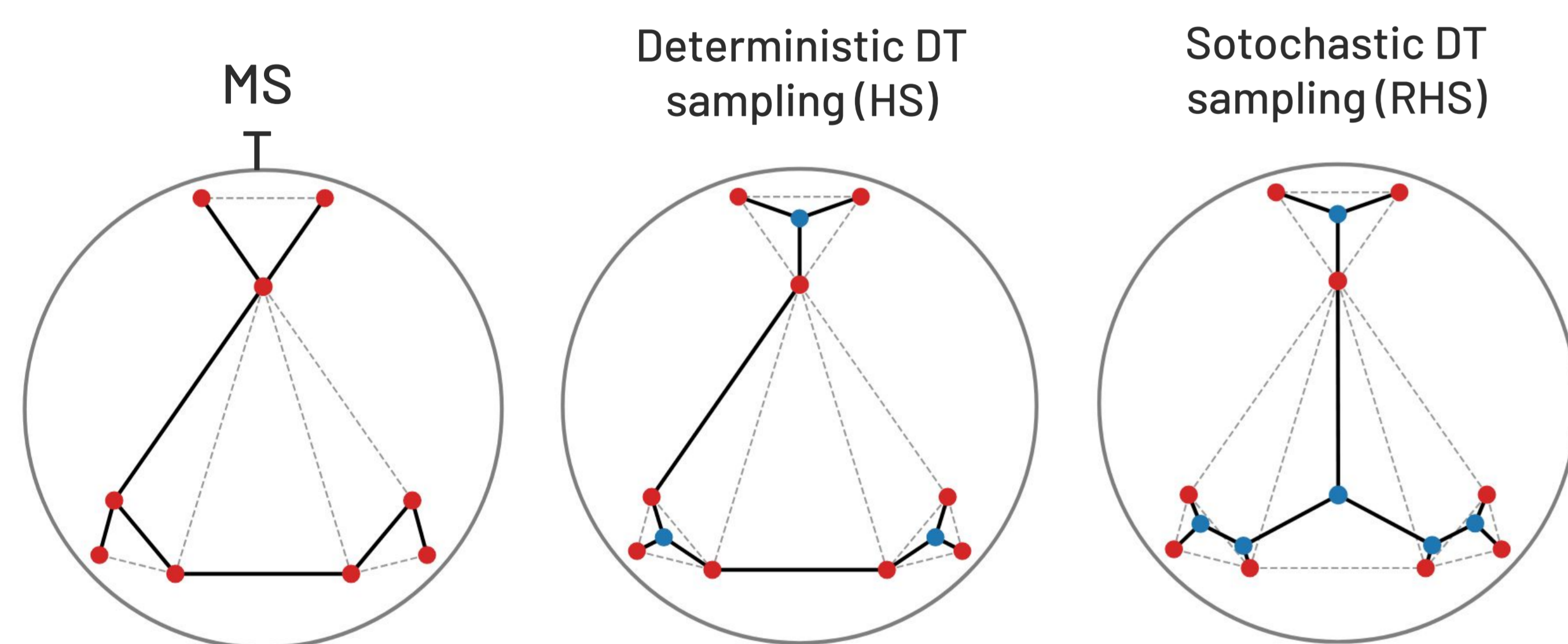


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## Introduction

**Hyperbolic geometry** is relevant to represent **hierarchical data**. We study the problem of constructing hyperbolic Steiner Minimal Trees (SMTs) in order to infer the hierarchy from a hyperbolic representation: **How to find the shortest tree connecting a set of points in a hyperbolic space?**

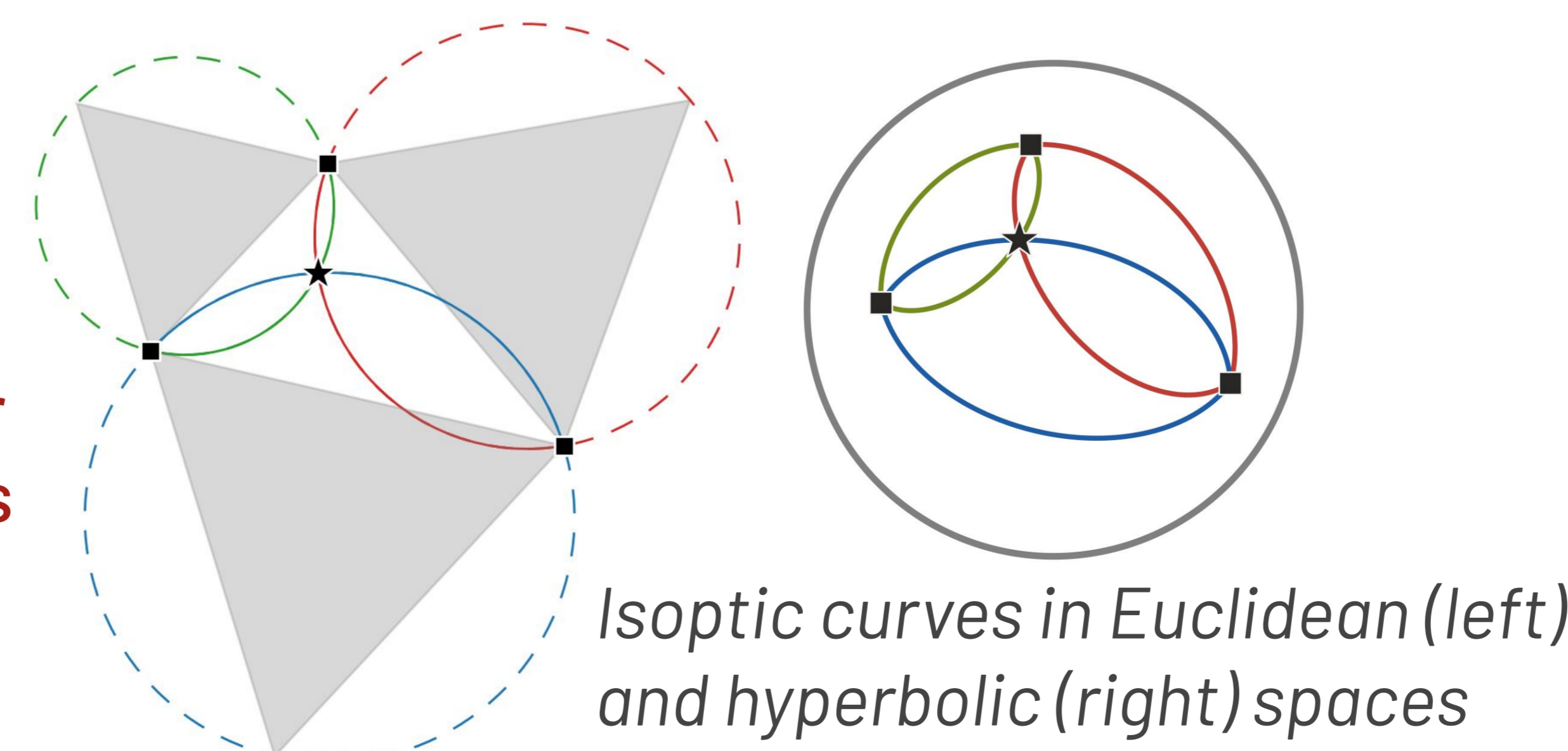
Exact SMT computation is NP-hard, and deterministic heuristics such as vanilla HyperSteiner get trapped in locally suboptimal topologies. We introduce **Randomized HyperSteiner (RHS)**, a stochastic Delaunay triangulation heuristic that incorporates randomness and refines candidate trees via Riemannian gradient descent optimization.



## Method

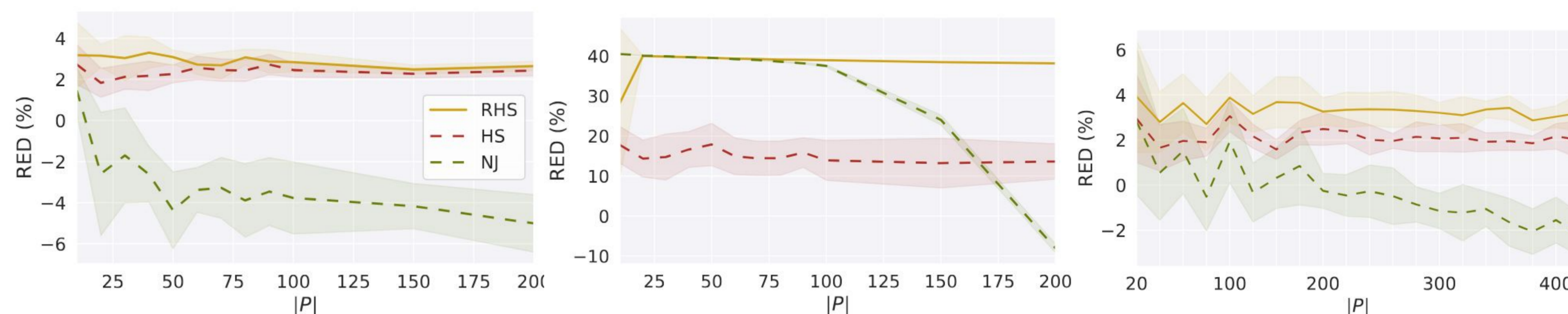
Given a terminal set embedded in the Klein-Beltrami model, RHS consists of:

- Computing the **hyperbolic Delaunay triangulation** of the current terminals and Steiner candidates.
- **Sampling triangles stochastically** to expand candidate Steiner points.
- **Refining** the resulting tree via **degree and angle conditions** (removing Steiner candidates and solving algebraic equations characterizing isoptic curves intersection), then **optimizing Steiner locations by Riemannian gradient descent**.

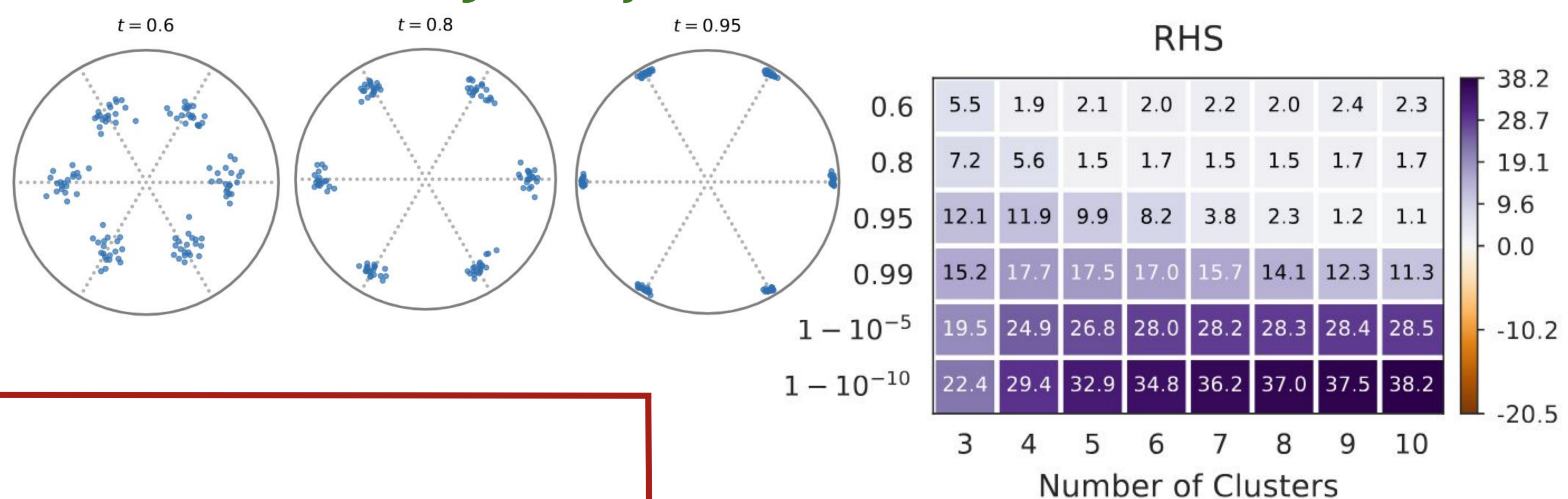


## Experiments

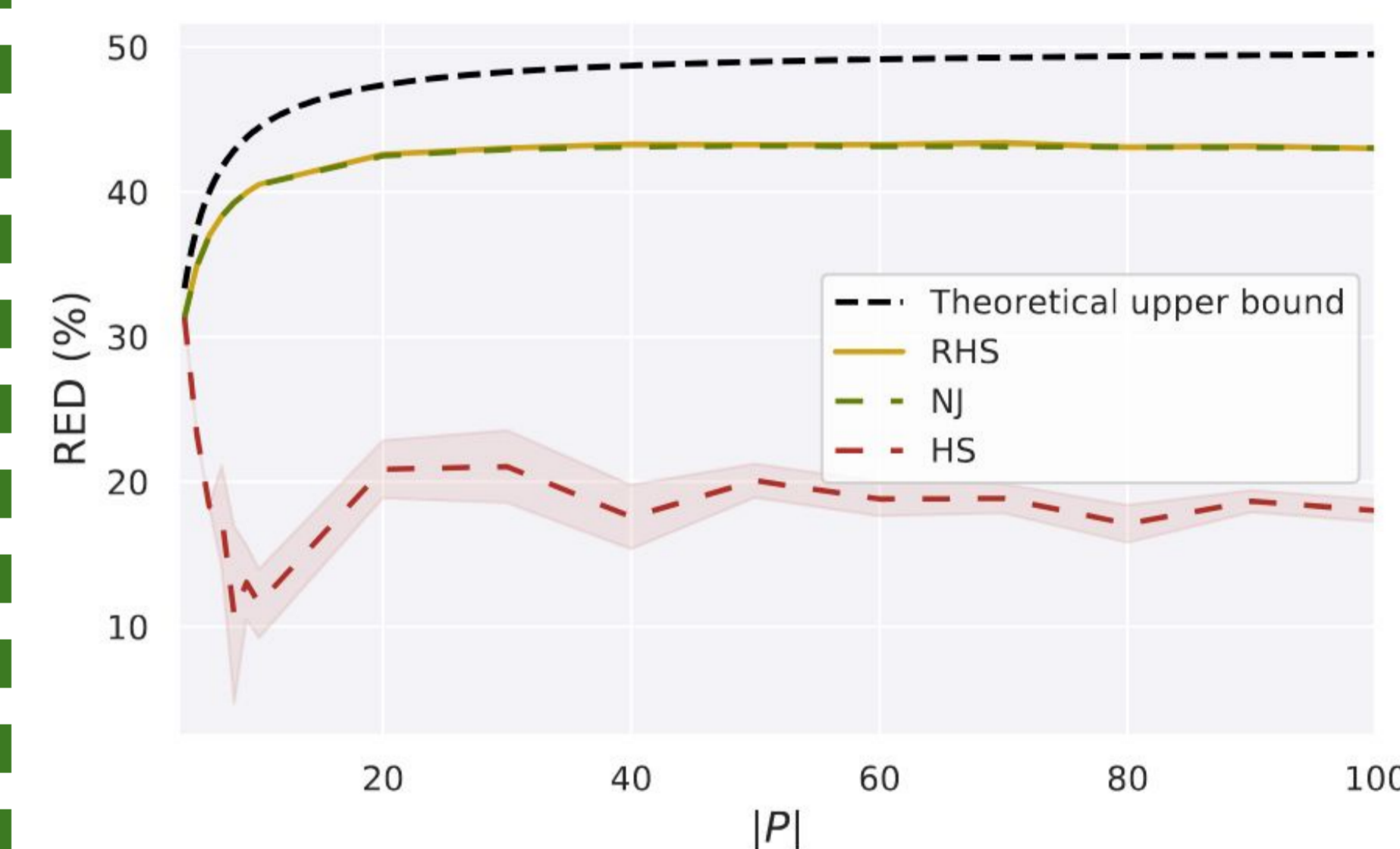
**1. RHS consistently outperforms vanilla HS, MST, and NJ** across synthetic and real data sets: centered Gaussian distribution (left), mixture of 10 Gaussians centered at a regular 10-gon near the boundary (middle), and *Planaria* biological data set (right).



**3. We sample 20 points per cluster from mixtures of  $d$  Gaussians centered at a regular  $d$ -gon, varying both the number of clusters  $d$  and a radial parameter  $t$ . Relative improvement over HS and NJ exceed respectively 33% and 43%. Reduction over MST increases drastically for  $t > 0.95$ , reaching results not achievable in Euclidean geometry.**



**2. RHS approaches the theoretical hyperbolic limit (50%), with reductions over MST up to 43.39% and relative improvement over deterministic HS exceeding 32% in extreme setting: regular  $|P|$ -gon near the boundary.**



## Conclusion

**RHS explores more candidate topologies, avoids myopic local choices, and can construct richer hierarchical structures with shorter total tree length.**

## Acknowledgements

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